A
Synopsis

On

"A Study of Some Special Functions with Fractional Integral Transforms"

Submitted to

Swami Ramanand Teerth Marathwada University,

Nanded-431606

For the degree of

Doctor of Philosophy in Mathematics

Under

Faculty of Science&Technology

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August 2021

1) Introduction:

Fractional Calculus

Fractional calculus is the field of mathematical analysis which deals with the investigations and applications of integrals and derivatives of arbitrary order, which we shall term as differ integral operators. It is also known by several other names such as Generalized Integral and Differential Calculus or Calculus of Arbitrary Order. Fractional calculus has been a topic of research for brilliant mathematical minds such as Fourier, Euler, Laplace and Liouville . It can be categorized as applicable Mathematics. During the last three decades scientists have found many supplications of fractional calculus in the field of Physics, Chemistry, Quantitative Biology, Engineering, Image and Signal Processing, Rheology, Diffusion and Transport Theory etc. (Podlubny, 1999, Kilbas et al., 2006 [8]). The fractional calculus has gained greater significance and popularity during the last four decades or so, mainly due to vast potential of its applications in various seemingly diverse fields of Science and Engineering, like Fluid Flow, Rheology, Diffusion, Oscillation, Anomalous Diffusion, Reaction-Diffusion, Turbulence, Diffusive Transport, Electric Networks, Polymer Physics, Chemical Physics, Electrochemistry of Corrosion, Reflection Processes in Complex System, Dynamical Process in selfsimilar and Porous Structures and others [1-8, 11, 13, 15].

The functional relations signify those relations which involve different type of special functions. But in the literature, specifically the relations between functions of hypergeometric nature and the logarithmic derivative of gamma function $\psi(z)$ have been designated as functional relations. This is an interesting area of research and many authors are working in this area. Over the last few decades several functional relations have been established by various researchers. Several attempts have also been made for their unification, most noteworthy being the contributions by Nishimoto, Saxena [20] and Chen, Srivastava [21]. Basically this is a part of fractional calculus as the functional relations are obtained by the application of fractional integral operators. Fractional calculus allows integral and derivative of any positive real order. It can be considered as a branch of mathematical analysis which deals with integrodifferential operators and equations where the integrals of convolution type and kernels of power law type are exhibited. It is closely related to the theory of pseudo-differential operators and special functions including new classes of functions associated with fractional calculus, integral transform, non-Gaussian, non-stochastic processes, control theory etc.[9,10,12,14,16-19].

Since Leibnitzs historical remark regarding consequences of fractional calculus, several attempts were made over the centuries, to formalize a definition which fulfils the criteria of analyticity of the function, similarity with ordinary

calculus, linearity and satisfies the laws of exponent of integration and differentiation. Many applications of Fractional Calculus can be found in Plasma Physics, Astrophysics, Fluid Dynamics, Non-linear Control Theory, Stochastic Dynamical System, Controlled Thermonuclear Fusion, Turbulence, Non-linear Biological Systems and Image Processing.

The idea of fractional is natural$\partial u/\partial x$ and $(\partial^2 u)/(\partial x^2)$exist, $(\partial^{1.5} u)/(\partial x^{1.5})$maybe exists too.

Special Functions Theory

is well known in mathematics for its apparent multiplicity of tricks and devices needed to obtain information about the subject matter. A study of the transformation theory and properties of the generalized hyper geometric function enable one to derive the properties of a large class of Special functions in a unified manner. There are three important aspects of the study of the generalized hyper geometric function which have attracted the attention of a majority of researches in this branch of analysis. The first aspect deals with the investigations leading to certain generalizations of the question of summing up , in terms of product of gamma function only, particular types of generalized hyper geometric function and their extensions. The second aspect comprises of the study of the interrelations that might exist between such function. The third aspect deals with the generalizations of these functions which have recently led to many remarkable applications in physics, Statistics, Number theory and Combinatory Analysis .

Definition of the Laplace transforms of fractional derivative:

The Laplace transform of the function $F(s)$ of the complex variable s defined as

$$\boldsymbol{F(s) = L\{f(t); s\} = \int_0^\infty e^{-st}\, f(t)dt}$$

Definition of Laplace transforms of the Riemann Liouville Fractional Derivative :

The Laplace transforms of the Riemann Liouvillefractional integral of order $p > 0$ definedas

$$\boldsymbol{L\{D_t^{-p} f(t); s\} = L\{D_t^{-p} f(t); s\} = s^{-p} F(s)}$$

Definition of the Mellin transforms of fractional derivative :

The Mellin integral transform F(s)of a functionf (t). which is defined in the interval $(0, \infty)$is

$$F(s) = M\{f(t); s\} = \int_0^\infty f(t)\, t^{s-1} dt$$

2) Review of Literature and Development in the Subject (Previous work done in the relevant areas):

Special functions found their utility in eigen value problems of quantum mechanics in 20th century. In 1907, the Gamma function was used by Barnes to develop a new theory of Gauss hypergeometric function 2F1. Then various generalizations of 2F1 were defined by Horn Kampé de Fériet, Mac Robert and Meijer. Being performing an important part in Physics and Engineering, its study and properties in theoretical way is very captivating.

In the year 1974, 1984 and 1989. Lots of simplifications have been worked for fractional integral operators by many research workers namely Kober[22] , Erdélyi [23], Sneddon [24]), Saxena [25], Koul [26], Raina and Kriyakova [27], Goyal and Jain , Al-Bassam [29], Weyl , Samko[30] and several others. Many authors to say Srivastava , Srivastava and Goyal , Srivastava and Owa , Manocha and Sharma, Kalla, Saigo, Saigo and Saxena, Gupta and Soni, Shrivastava, Kilbas and Saigo, Saxena and Saigo, Saxena and Nishimoto, Chaurasia and Gupta, Chaurasia and Godika Chaurasia and Singhal , Chaurasia and Shekhawat [28],etc.

3) Objective of Research/ Proposed Hypothesis:

The purpose of this study is we will establish the theorem by obtaining the integral which is the result of product of certain general class of polynomials and the special functions. The results will be established by certain integrals using Special functions, Special functions of several complex variables and general polynomials. we will obtain the solution of partial differential equations concerning to the problem of Angular displacement in a shaft. It has important role in the field of Physics, Applied Mathematics and Mathematical Analysis. The Special functions, general class of polynomials have been used to obtain certain solutions.

4) Methodology to be adopted:

We use different techniques to obtain the results such as:

1) The Aleph Function
2) Generalized Hypergeometric Function
3) The Generalized Polynomials
4) Laplace Transform.
5) Mellin Transform etc.

5) Importance of Study/ Society Applications:

➢ International Status:

Many mathematicians are working in this field, some of them are A. Secer (***Turkey***), A. Atangana (***South Africa***), Y. Povstenko (**Poland**), **M.B_lasik** (**Poland**), G. D. Medina (**Argentina**), L. G. Romero (**Argentina**),**F. Mainardi** (**Italy**), **R. Garra**(**Italy**), A. Ebaid (**Saudi Arabia**), E. Bonyah (**South Africa**),**Li Ma** (***China),*** K. S. Nisar(**Saudi Arabia**), G. Wang (***China***), H. Habenom(***Ethiopia***), **J. Choi** (**Republic of Korea**), E. Bonyah (**Ghana**), A. Oli(***Ethiopia***), etc.

➢ National Status:

SomeoftheIndianMathematiciansarealsoworkingonsameline.FewofthemareDinesh Kumar,(*Jodhpur*),S. D. Purohit (*Rajasthan*), M.Chand(Bathinda), P. Agarwal (**Jaipur**), M. Rakshit (**Bathinda**), Gurmej Singh(**Jhunjhunu**), S. D. Purohit (**Kota**), J. Singh (**Rajasthan**), **M. Bansal** (**Rajasthan**), **P. Harjule** (**Rajasthan**), J. C. Prajapati (**Gujarat**), R. K. Parmar (**Bikaner**), S. D. Purohit (**Rajasthan**), Owais Khan (**Aligarh**), ***A. Bhargava***(**Jaipur**), etc.

➢ Significance of Study:

Many problems of fractional Calculus and fractional Integral Transforms and application of Special functions play a major role in the mathematical modeling of many fields such as physical, biological phenomena and engineering sciences that necessarily take into account the effect of real world problems.

6) Proposed Workplan / Formulation and Structure of Study:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First Year	Study of the background required for the research topic, like Special functions and its properties, **Fractional Calculus,** Some fractional **Integral Transforms with Applications** etc.	Good command on the subject and clear the coursework.
2	Second Year	Collecting the papers and analyze them.	Try to get some new result based on the previous work.
3	Third Year	Work on some problems to be included in the thesis.	Submit at least Three papers.
4	Fourth Year	Publishing some papers and submitting the thesis.	Some more papers and thesis submission.

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